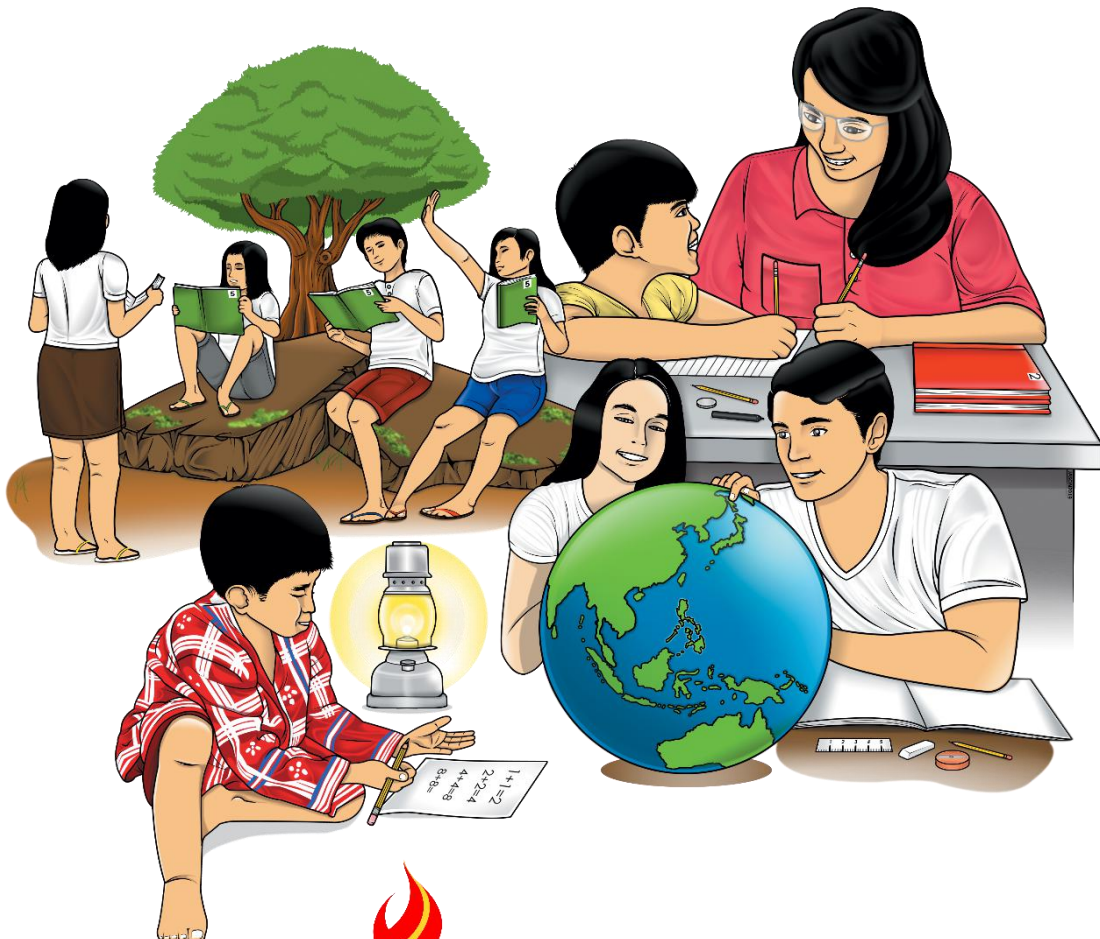


Senior High School

General Mathematics

Quarter 2 – Module 4:

Simple and General Annuities



General Mathematics – Senior High School
Alternative Delivery Mode
Quarter 2 – Module 4: Simple and General Annuities
First Edition, 2021

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General Mathematics

Quarter 2 – Module 4:

Simple and General Annuities

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



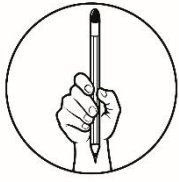
What I Need to Know

One of the dreams of every Filipino family is to purchase a house where they can comfortably live. However, most often they cannot afford to purchase such property in cash. One of the options to realize their dreams is to purchase it on an installment basis. In this situation, the concept of annuities is being used.

In this learning module, learners should be able to investigate and analyze problems involving simple and general annuities. This module was designed and written with you in mind. It is here to help you identify and distinguish simple and general annuities.

After going through this module, you are expected to:

1. define terms that are related to simple and general annuities;
2. illustrates and distinguishes simple and general annuities; and
3. represent annuities to real-life situations.



What I Know

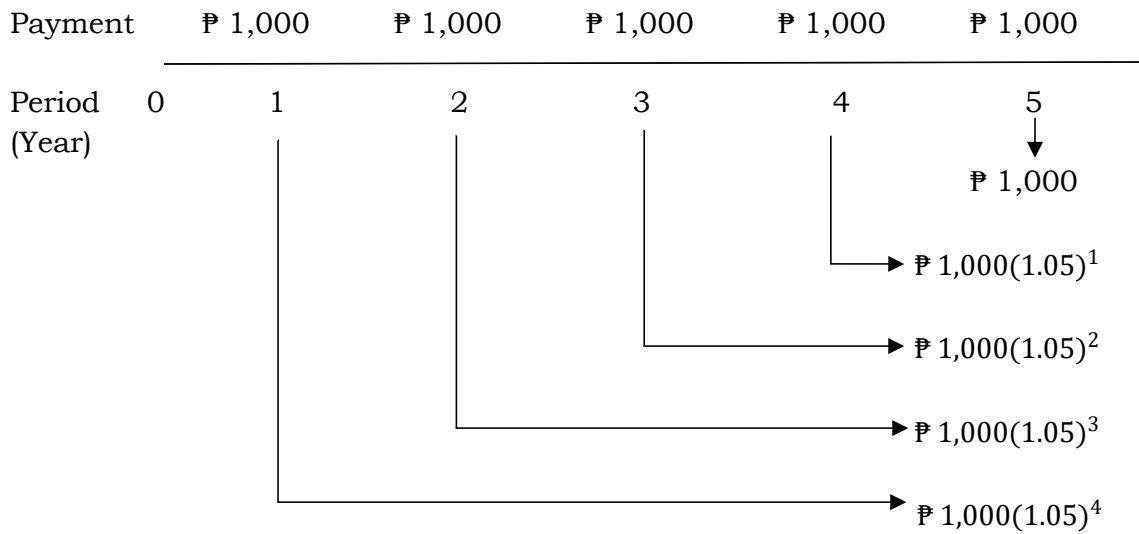
Let's find out how far you might already know about this topic! Please take this challenge! Have Fun!

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What is referred to as a fixed sum of money paid to someone at regular intervals, subject to a fixed compound interest rate?
 - a. compound interest
 - b. simple interest
 - c. annuity
 - d. annuity certain
2. What type of annuity is illustrated when the interest conversion or compounding period is equal or the same as the payment interval?
 - a. annuity certain
 - b. annuity uncertain
 - c. simple annuity
 - d. general annuity
3. What type of annuity is illustrated when the interest conversion or compounding period is unequal or not the same as the payment interval?
 - a. annuity certain
 - b. annuity uncertain
 - c. simple annuity
 - d. general annuity
4. What do you call the fixed payment and interest over a specified time?
 - a. annuity payment
 - b. interest payment
 - c. due payment
 - d. general payment
5. Which is an example of annuities?
 - a. interest
 - b. rent
 - c. bills
 - d. savings
6. What do you call the type of annuity in which the payments are made at the end of each payment interval?
 - a. annuity certain
 - b. contingent annuity
 - c. ordinary annuity
 - d. general annuity
7. What do you call to the type of annuity which payments begin and end at definite times?
 - a. contingent annuity
 - b. ordinary annuity
 - c. simple annuity
 - d. annuity certain

8. What do you call the type of annuity in which the payments extend over an indefinite length of time?
- a. contingent annuity
 - b. annuity certain
 - c. general annuity
 - d. ordinary annuity
9. A payment made at the end of each month for money borrowed that charges 1.05% interest compounded quarterly. What type of annuities is being represented?
- a. simple
 - b. general
 - c. ordinary
 - d. annuity due
10. What type of annuity is represented by a deposit of ₱ 6,000.00 that is made at the end of every three months to an account that earns 5.6% interest compounded quarterly?
- a. simple
 - b. general
 - c. ordinary
 - d. annuity due
11. Which of the following situations is an example of a simple annuity?
- a. ₱ 2,000 deposited every six months for 2 years at 8% per year compounded monthly
 - b. ₱ 2,000 deposited every month for 2 years at 8% per year compounded annually
 - c. ₱ 2,000 deposited every month for 2 years at 8% per year compounded monthly
 - d. ₱ 2,000 deposited every quarter for 2 years at 8% per year compounded monthly
12. Which of the following situations is an example of a general annuity?
- a. ₱ 5,000 deposited every month for 5 years at 8% per year compounded annually
 - b. ₱ 5,000 deposited every six months for 5 years at 8% per year compounded semi-annually
 - c. ₱ 5,000 deposited every three months for 5 years at 8% per year compounded quarterly
 - d. ₱ 5,000 deposited every year for 5 years at 8% per year compounded annually

For numbers 13-15. Given the cash flow of an annuity below.



13. What is the rate of interest?

- a. 1.05%
- b. 5%
- c. 0.05%
- d. 0.5%

14. What is the term of the payment?

- a. 5 years
- b. 4 years
- c. 6 years
- d. 1000 years

15. What is the sum of all the payments of the given annuity?

- a. ₱ 5,000
- b. ₱ 5,455.82
- c. ₱ 5,225.36
- d. ₱ 5,525.63

Lesson

1

Simple and General Annuities

This lesson will tackle first the definition and types of annuities and the difference between simple and compound annuities. There are types of annuity according to correspondence of payment intervals with interest periods

Simple Annuity - the payment interval is also the same as the interest period.

General Annuity refers to an annuity where the length of the payment interval is not the same as the length of the interest compounding period



What's In

REMEMBER ME

As the saying goes, "A person who does not remember where he came from will never reach his destination". This saying is very popular, passing from one generation to another generation. For you to remember the previous lesson, here are some exercises to refresh your mind.

Activity 1

Answer each of the following.

1. If ₱40,000 is invested for 6 years at 5% compounded quarterly.
 - A. Give the value of the following using the formula $A = P(1 + i)^n$ where $i = \frac{r}{k}$ and $n = kt$
 1. $P =$ _____
 2. $r =$ _____
 3. $i =$ _____
 4. $n =$ _____

- B. Find the compound amount.

Solution:

Answer:

C. Find the compound interest

Solution:

Answer:

2. ₱ 20,000 is invested for 15 years at 5 % compounded semi-annually.

A. Give the value of each variable using the formula $A = P(1 + i)^n$

where $i = \frac{r}{k}$ and $n = kt$

1. P= _____
2. r= _____
3. i= _____
4. n= _____

B. Find the compound amount.

Solution:

Answer:

C. Find the compound interest.

Solution:

Answer:



What's New

WISE DECISION

Mr. and Mrs. De Dios are planning to buy a new house despite of their limited budget. They seek advice from a bank for them to produce enough amount for the down payment of the house they have chosen.

According to the bank if ₱ 20,000 will be invested at the end of each year for 5 years in an account that pays interest at 10% compounded annually the couple will be able to have the amount for down payment.

If there are 3 houses for sale offering a 10% down payment of the price which among these 3 houses is the best choice?

House A – ₱850,740

House B – ₱1,221,020

House C – ₱2,110,000

For you to be able to determine the total amount of investment for the period of 5 years, you need to complete the table below.

Calculate the amount to which the first to the fourth period ₱ 20,000.00 will grow using the compound interest formula.

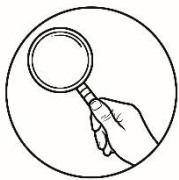
Period	Amount in Exponential Form	Amount in Pesos
1	$₱20,000(1.01)^4$	₱ 29,282 [No space]
2		
3		
4		
5	₱20,000	₱ 20,000
Total		=

Note: The ₱ 20,000 deposited at the end of the first year will draw interest for 4 years, so it will amount to $₱20,000(1.01)^4$. The ₱20,000 deposited at the end of the second year will draw interest for 3 years so it will amount to $₱20,000(1.01)^3$, and so on.

1. What pattern will be reflected in the 2nd column of the table? Discuss.

2. Using the pattern found in number 1, calculate the sum of the amounts in the third column.

3. Given the answer in number 2, what will be the best house suited to the investment made by Mr. and Mrs. De Dios? Explain.



What is It

The activity in the **What's New** involves fixed payment/investment for each period (₱ 20,000 at the end of each year) and fixed compound interest rate over a specified time; that in this case is 10% compounded annually for 5 years.

If the payment for each period is fixed and the compound interest rate is fixed over a specified time the payment is called an **annuity payment**. Accounts associated with streams of annuity payments are called **annuities**.

Annuity - a sequence of payments made at equal (fixed intervals or periods of time).

The following are examples of annuities:

- Rental payment
- Monthly pensions
- Monthly payment for car loan
- Educational plan

Annuities may be classified in different ways, as follows.

	Annuities	
According to payment interval and interest period	Simple Annuity - an annuity where the payment intervals is the same as the interest period	General Annuity - an annuity where the payment intervals is not the same as the interest period
According to time of payment	Ordinary Annuity (or Annuity Immediate) - a type of annuity in which the payments are made at the end of each payment interval	Contingent Annuity - an annuity in which the payments extend over an indefinite (or indeterminate) length of time
According to duration	Annuity Certain - an annuity in which payments begin and end at definite times	Contingent Annuity - an annuity in which the payments extend over an indefinite (or indeterminate) length of time

In the activity, the payment is made at the end of each period. Such annuity is called an **ordinary annuity**.

Each payment in an annuity is called the **periodic payment (R)**.

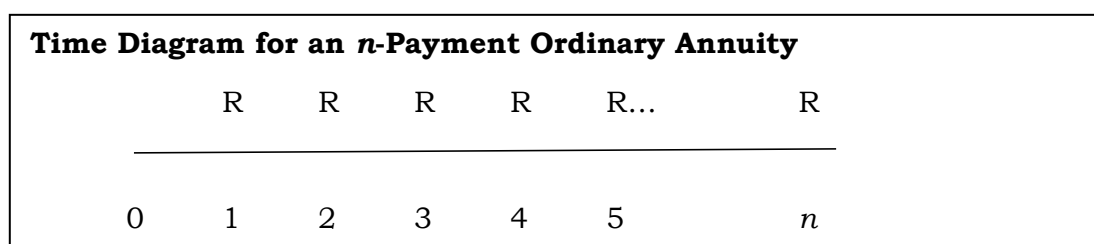
The time between the successive payments dates of an annuity is called the **payment interval**.

The time between the first payment interval and last payment interval is called **term of the annuity (t)**.

The sum of the future values of all the payments to be made during the entire term of the annuity is the **future value or the amount of an annuity (F)**.

The sum of the present values of all payments to be made during the entire term of the annuity is called the **present value of n annuity (P)**.

Annuities may be illustrated using a time diagram. The time diagram for an ordinary annuity is given below.



Simple Annuity - the payment interval is also the same as the interest period.

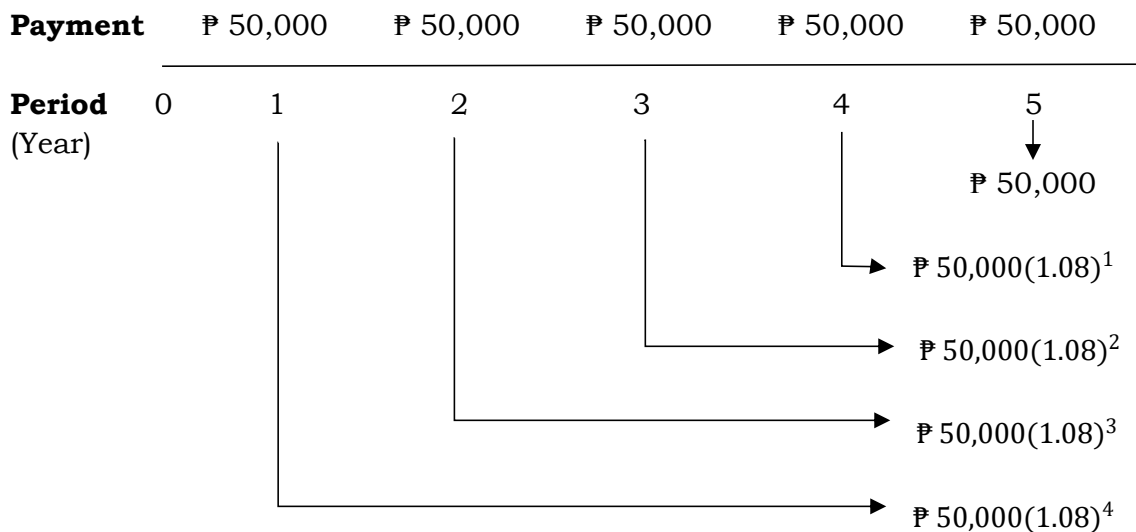
Example 1. ₱ 50,000 deposited every year for 5 years at 8% per year compounded annually.

Solution: Notice that ₱ 50,000 was deposited every year and it is compounded annually. Since the compounding period is similar to the payment interval, then this is a type of simple annuity.

The cash flow of the given situation can be illustrated in the time diagram below:

Payment	₱ 50,000	₱ 50,000	₱ 50,000	₱ 50,000	₱ 50,000	
Period	0	1	2	3	4	5

The future value of all the payments at the end of term (t=5).



Add all the future values obtained from the previous step.

₱ 50,000.00	=	₱ 50,000
₱ 50,000(1.08) ¹	=	₱ 54,000
₱ 50,000(1.08) ²	=	₱ 58,320
₱ 50,000(1.08) ³	=	₱ 62,985.60
₱ 50,000(1.08) ⁴	=	<u>₱ 68,024.45</u>
Total		₱ 293,330.05

General annuity refers to an annuity where the length of the payment interval is not the same as the length of the interest compounding period.

Examples of General annuity:

1. Monthly installment of a car, lot or house with an interest rate that is compounded annually.
2. Paying a debt semi-annually when the interest is compounded monthly.

Example 2: Find the amount of annuity of ₱700 every 6 months ($\frac{1}{2}$ year) for 12 years if interest is 6% compounded monthly.

Solution: Here, the payment interval ($\frac{1}{2}$ year) is different than the interest period (annual). This is a general annuity.

We must match the interest period to the payment interval.

We must find the semi-annual rate that is equivalent to 6%, compounded monthly.

Step 1: Using the formula $A = P(1 + i)^n$, find the value of ₱1 invested at 6%, compounded monthly after 1 year.

$$A = 1(1 + 0.005)^{12} \quad \text{Note: } I = 0.06/12 = 0.005, n = 12$$
$$= 1.061677812$$

Step 2: Let the equivalent $\frac{1}{2}$ year rate be $i\%$ (Note the equivalent yearly rate would be $2i\%$). Now find the value of ₱1 invested at $i\%$ per $\frac{1}{2}$ year after 1 year.

$$A = 1(1 + i)^2 \quad \text{Note: } n = 2, \text{ the number of times interest is compounded}$$

Step 3: These two amounts must be equal. Hence

$$(1 + i)^2 = 1.061677812$$

$$1 + i = \sqrt{1.061677812} = 1.030377509$$

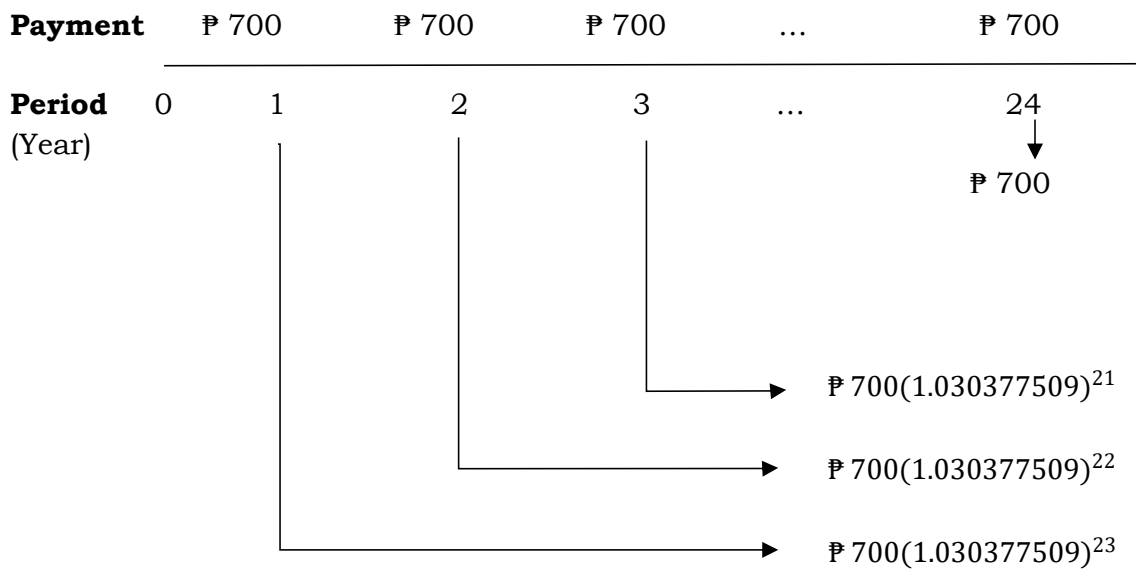
$$i = 0.030377509 \text{ or } 3.0377509\% \text{ (per } \frac{1}{2} \text{ year)}$$

Therefore, the equivalent $\frac{1}{2}$ year rate is 3.0377509%

Now, we can illustrate the cash flow:

Payment		₱ 700	₱ 700	₱700 ...	₱ 700
Period	0	1	2	3 ...	24

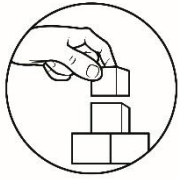
The future value of all the payments at the end of term (t=24).



Add all the future values obtained from the previous step.

$$= \text{₱ } 700 + \text{₱ } 700(1.030377509) + \text{₱ } 700(1.030377509)^2 + \dots + \text{₱ } 700(1.030377509)^{23}$$

$$= \text{₱ } 24,212.83$$



What's More

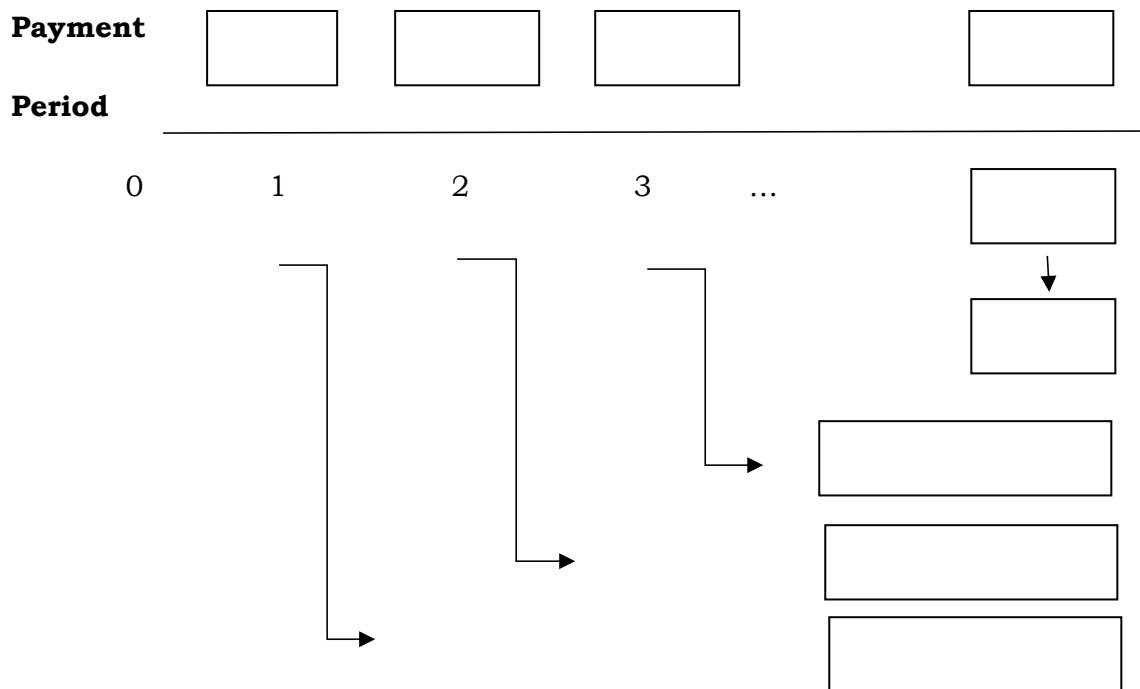
Activity 1.1

Read each problem carefully and tell whether each of the given information describes a simple annuity or general annuity. Complete the diagram that follows. Have Fun!

- a. Monthly payments of ₱ 3,000 for 4 years with interest rate of 3% compounded monthly.

Type of Annuity: _____

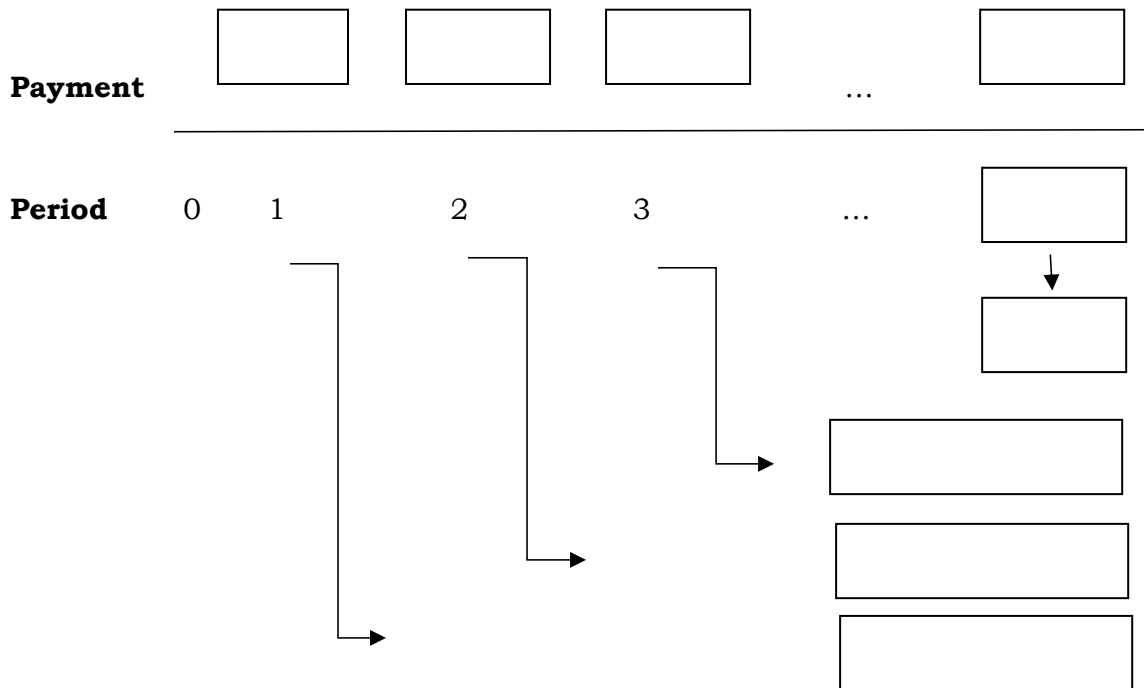
Class flow in the time diagram:



- b. Quarterly payment of ₱ 5,000 for 10 years with interest rate of 2% compounded semi-annually.

Type of Annuity: _____

Diagram:



Activity 1.2

Identify the type of annuity in the given situations below then show the illustration of the cash flow.

- a. Monthly payments of ₱ 2,000 for 5 years with interest rate of 12% compounded annually.

- b. Yearly payment of ₱15,000 for 10 years with interest rate of 8% compounded annually.



What I Have Learned

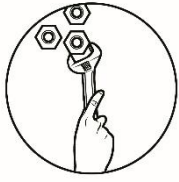
A. Please read the sentences carefully and fill in the missing word/s by writing your answer on the line/s provided.

1. A sequence of payments made at equal (fixed) intervals or periods of time is called _____.
2. Each payment in an annuity is called _____.
3. The time between the successive payments dates of an annuity is called the _____.
4. In simple annuity the payment interval is also the same as the _____.
5. In general annuity the interest conversion or compounding period is _____ with the payment interval.

B. Give at least three examples of real-life situations in your locality that shows annuity.

C. Enumerate and describe the different types of annuities.

D. Being aware with the two types of annuities help you in different transactions that you will encounter in the future. What are the advantages and disadvantages of these annuities?



What I Can Do

Read and analyze the situation below then answers the question given.

Your mom asked your opinion if she will be joining the cooperative in her office. She wants to know the amount of money she will be receiving after 6 months and wanted to buy something in December. The cooperative wants her to contribute ₱1,000 per month beginning in June 2020 which will earn 3% compounded monthly. How much will be the future value of your mom's contribution at the end of December 2020?

Tasks:

You need to prepare a report showing a cash flow diagram on the total amount of money your mother will earn at the end of December. At the end of your report, write a conclusion stating your opinion to help your mom to decide.

Cash Flow Diagram

Conclusion:

Rubrics:

5-States a conclusion with complete and appropriate justification, based on a reasonable interpretation of the data.

4-States a conclusion with enough justification, based on a reasonable interpretation the data.

3-States a conclusion with some justification, based on a reasonable interpretation of the data.

2-States a conclusion on a reasonable interpretation of the data.

1-The conclusion is based on an unreasonable interpretation of the data.



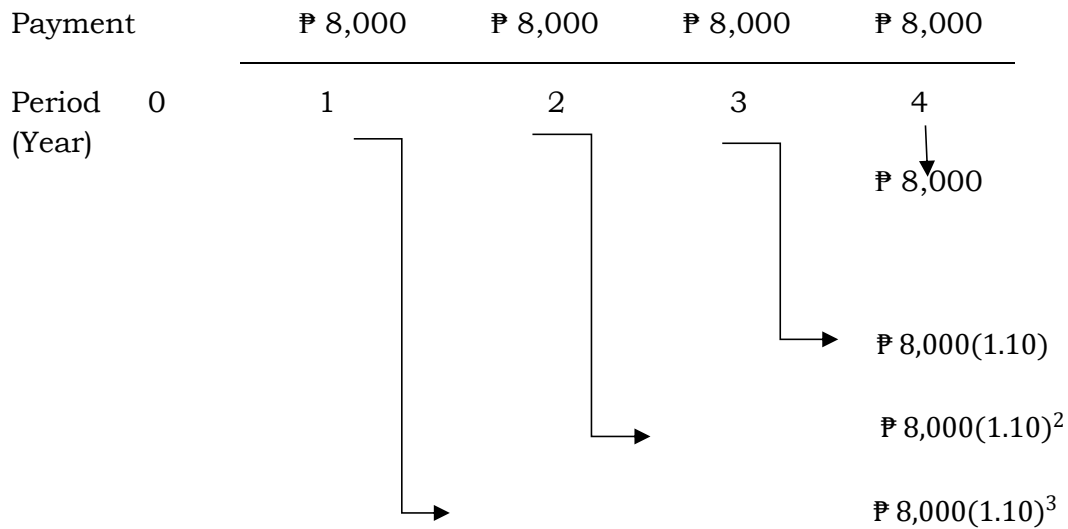
Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

- Which of the following refers to the fixed sum of money paid to someone at regular intervals and subject to a fixed compound interest?
 - annuity
 - simple interest
 - compound interest.
 - annuity certain
- If the interest conversion or compounding period is equal or the same with the payment interval, what type of annuity is illustrated?
 - annuity certain
 - annuity uncertain
 - simple annuity
 - general annuity
- If the interest conversion or compounding period is unequal or not the same as the payment interval, which type of annuity will be used?
 - annuity certain
 - general annuity
 - simple annuity
 - annuity uncertain
- Which of the following is NOT an example of annuities?
 - pension
 - educational plan
 - car loan
 - deposit
- Each payment in an annuity.
 - interest payment
 - periodic payment
 - loan payment
 - cash payment
- The time between the successive payment dates of annuities.
 - payment interval
 - payment due
 - periodic payment
 - term
- The interval between the beginning of the first payment period and the end of the last period.
 - period
 - due
 - term
 - interval
- The sum of all the payments to be made during the entire term of the annuity.
 - future value
 - present value
 - loan value
 - interest value

9. What type of annuity is represented if the payment is made at the end of each month for money borrowed that charge 0.15% interest compounded quarterly?
- a. simple
 - b. general
 - c. ordinary
 - d. annuity due
10. What type of annuity is represented by a deposit of ₱10,000.00 that is made at the end of every three months to an account that earns 2.6% interest compounded quarterly?
- a. simple
 - b. general
 - c. ordinary
 - d. annuity due
11. Which of the following situations is NOT an example of simple annuity?
- a. ₱1,500 deposited every month for 15 years at 10% per year compounded annually
 - b. ₱1,500 deposited every month for 15 years at 10% per year compounded monthly
 - c. ₱1,500 deposited every six months for 15 years at 10% per year compounded semi-annually
 - d. ₱1,500 deposited every three months for 15 years at 10% per year compounded quarterly
12. Which of the following situations is an example of a general annuity?
- a. ₱5,000 deposited every month for 5 years at 8% per year compounded annually
 - b. ₱5,000 deposited every six months for 5 years at 8% per year compounded semi-annually
 - c. ₱5,000 deposited every three months for 5 years at 8% per year compounded quarterly
 - d. ₱5,000 deposited every year for 5 years at 8% per year compounded annually

For numbers 13-15. Given the cash flow of an annuity below.



13. What is the rate of interest?

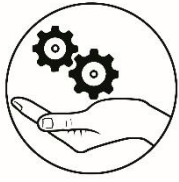
- a. 1.10%
- b. 10%
- c. 0.10%
- d. 0.01%

14. What is the term of the payment?

- a. 10 years
- b. 4 years
- c. 10 months
- d. 4 months

15. What is the sum of all the payments of the given annuity?

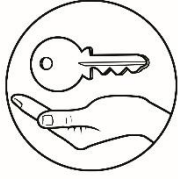
- a. ₱ 30,218
- b. ₱ 31,718
- c. ₱ 32,000
- d. ₱ 37,128



Additional Activities

1. Why is it that many Filipinos do not save or save very little? Research the top five external factors or barriers to saving. Your answers should be supported by recent data. Cite your sources for the data gathered.

2. Choose one topic from the choices shown below. Ask someone how the payment/ investment has been made. Make research on some strategies on how to reduce or gain interest in the chosen topic.
 - a. Insurance
 - b. Time deposit
 - c. Educational plan
 - d. Housing Loan
 - e. Retirement Plan
 - f. Investment



Answer Key

Assessment

1. a
2. c
3. b
4. d
5. b
6. a
7. c
8. a
9. b
10. a
11. a
12. a
13. b
14. b
15. d

What's More

Activity 1.1

1. Simple Annuity
 $\text{P}3,000$ $\text{P}3,000$ $\text{P}3,000$... $\text{P}3,000$
 1 2 3 ... 48
 $\text{P}3,000$
 $\text{P}3,000(1.0025)^{45}$
 $\text{P}3,000(1.0025)^{46}$
 $\text{P}3,000(1.03)^{47}$

2. Compound Annuity
 $\text{P}5,000$ $\text{P}5,000$ $\text{P}5,000$... $\text{P}5,000$
 1 2 3 ... 40
 $\text{P}5,000$
 $\text{P}5,000(1.0066227)^{37}$
 $\text{P}5,000(1.0066227)^{38}$
 $\text{P}5,000(1.0066227)^{39}$

Activity 1.2

1. Compound Annuity
 $\text{P}2,000$ $\text{P}2,000$ $\text{P}2,000$... $\text{P}2,000$
 1 2 3 ... 60
 $\text{P}2,000$
 $\text{P}2,000(1.009488)^{57}$
 $\text{P}2,000(1.009488)^{58}$
 $\text{P}2,000(1.009488)^{59}$

2. Simple Annuity
 $\text{P}1,500$ $\text{P}1,500$ $\text{P}1,500$... $\text{P}1,500$
 1 2 3 ... 10
 $\text{P}3,000$
 $\text{P}3,000(1.08)^7$
 $\text{P}3,000(1.08)^8$

What I Know

1. c
2. c
3. d
4. a
5. b
6. c
7. d
8. a
9. b
10. a
11. c
12. a
13. b
14. a
15. d

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